system of equations is 3 h 40 min while the time of calculation of the temperature at one internal point is 45 sec.

The method developed was tested by comparing the results of the calculation with experimental data obtained by blowing through blades with combined cooling (convective + film cooling) at the hot wall. A description of the experimental subject is given in [7].

The measured and calculated temperatures were compared for the following operating parameters: $T_g^* = 905.7°K$, $P_g^* = 1.37 \cdot 10^5$ N/m²; $T_{a.in}^* = 373°K$; $\bar{G}_a = G/G_g = 3\%$.

The results of temperature measurements at the surface of the test blade are shown by points in Fig. 3. Curve 1 is a calculation by the two-dimensional method of [2] with the perforations combined in one cross section; 2) by the same method in a cross section where perforations are absent; 3) calculation of the three-dimensional temperature field in a characteristic element of the blade with allowance for the spatial distribution of the perforations.

It is seen from the graphs that in the middle part of the profile, where the character of the temperature field is close to two-dimensional, the results of the calculations by the two-dimensional and three-dimensional theories differ little. In those places where the temperature field has a clearly expressed three-dimensional character (at the edges due to the presence of perforations) the calculations by the two-dimensional theory can lead to considerable errors (up to 20% at the inlet edge in the example under consideration).

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HEAT TRANSFER OF A VERTICAL CYLINDER BY FREE CONVECTION AND RADIATION

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UDC 536.244

The effect of radiation on free convective heat liberation from the surface of a vertical cylinder located in a transparent medium is studied. It is shown that the radiative component of thermal flux equalizes the surface temperature.

Calculation of the thermal regimes of radio electronic devices requires study of heat transfer from high temperature elements to the surrounding medium. In calculating heat liberation from the surfaces of bodies of semiconductor devices, thermoresistors, microconductors, etc., it is necessary to consider the effect of not only transverse curvature on heat transfer, but also the interaction of various forms of heat transfer. Of special interest in electronics is heat transfer to an immobile medium by free convection and radiation. Existing studies of this problem have considered the case of a plane surface and have mainly been performed by approximate methods [1-4].

We will consider free motion of a viscous incompressible gas with constant physical properties in a boundary layer near a vertical cylinder. The gas is considered optically transparent and we neglect the processes of radiation emission, absorption, and scattering. The

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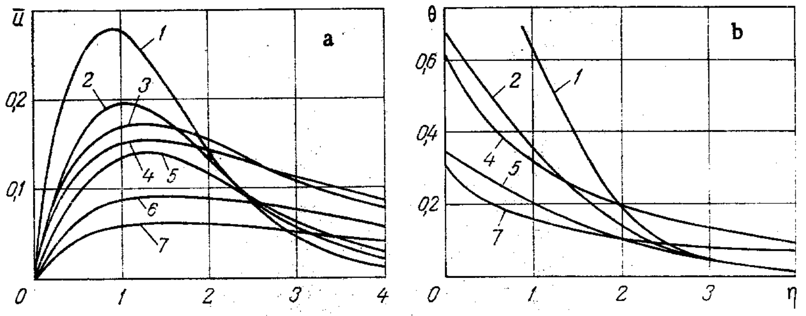

Fig. 1. Dimensionless velocity (a) and temperature (b) profiles in boundary layer: 1) $\xi = 0$ (plate, q_w = const); $\xi = 0.2$; 2) $S = 0$, $B = 1$ (plate); 3) $S = 5$, $B = 1$; 4) $S = 10$, $B = 0$; 5) $S = 0$, $B = 1$ (plate); 6) $S = 5$, $B = 1$; 7) $S = 10$, $B = 0$.

gravitational force is defined in the Boussinesq approximation, and viscous dissipation is absent. In this case the equations for momentum transfer, energy, and flow continuity are written in the form

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = \frac{\nu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) + g\beta(T - T_\infty), \tag{1}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r} = \frac{a}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right), \tag{2}$$

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial r}(rv) = 0. \tag{3}$$

We assume that the cylinder surface emits into the surrounding medium a constant thermal flux q_w, which is carried off as convective-conductive and radiative components. The latter are defined by local surface temperature and the surface radiation properties and are not independent. We consider the cylinder surface to be a gray diffusion radiator with emissivity ε. Then for the radiant component of thermal flux on the wall we may use the Stefan–Boltzmann law and write boundary conditions in the form

$$\begin{gathered} u = v = 0,\ q_w = -\lambda\left(\frac{\partial T}{\partial r}\right)_w + \sigma\varepsilon(T_w^4 - T_\infty^4) \text{ for } r = R, \\ u = 0,\ T = T_\infty \text{ for } r \to \infty. \end{gathered} \tag{4}$$

Commencing from continuity equation (3), we introduce the flow function, and transform in Eqs. (1)-(4) to the variables

$$\begin{gathered} \eta + (\mathrm{Gr}_x^*/5)^{1/5}\frac{r^2 - R^2}{2xR},\ \xi = (\mathrm{Gr}_x^*/5)^{-1/5}\sigma\varepsilon T_\infty^3 x/\lambda, \\ \psi = 5\nu R(\mathrm{Gr}_x^*/5)^{1/5} f(\xi, \eta),\ T - T_\infty = \theta(\xi, \eta)\frac{q_w x}{\lambda}(\mathrm{Gr}_x^*/5)^{1/5}, \end{gathered} \tag{5}$$

where $\mathrm{Gr}_x^* = g\beta q_w x^4/(\lambda\nu)$, which are a generalization of the planar plate variables of [1] to the axisymmetric case, or to variables for a vertical cylinder in the presence of radiation.

In the variables of Eq. (5), the problem (1)-(4) takes on the form

$$\begin{gathered} (1 + S\eta\xi)\frac{\partial^3 f}{\partial\eta^3} + 4f\frac{\partial^2 f}{\partial\eta^2} - 3\left(\frac{\partial f}{\partial\eta}\right)^2 + S\xi\frac{\partial^2 f}{\partial\eta^2} + \theta = \\ = \xi\left(\frac{\partial f}{\partial\eta}\frac{\partial^2 f}{\partial\eta\partial\xi} - \frac{\partial^2 f}{\partial\eta^2}\frac{\partial f}{\partial\xi}\right), \end{gathered} \tag{6}$$

$$\begin{gathered} \frac{1}{\mathrm{Pr}}(1 + S\eta\xi)\frac{\partial^2\theta}{\partial\eta^2} + 4f\frac{\partial\theta}{\partial\eta} - \theta\frac{\partial f}{\partial\eta} + \frac{S\xi}{\mathrm{Pr}}\frac{\partial\theta}{\partial\eta} = \\ = \xi\left(\frac{\partial f}{\partial\eta}\frac{\partial\theta}{\partial\xi} - \frac{\partial f}{\partial\xi}\frac{\partial\theta}{\partial\eta}\right), \end{gathered} \tag{7}$$

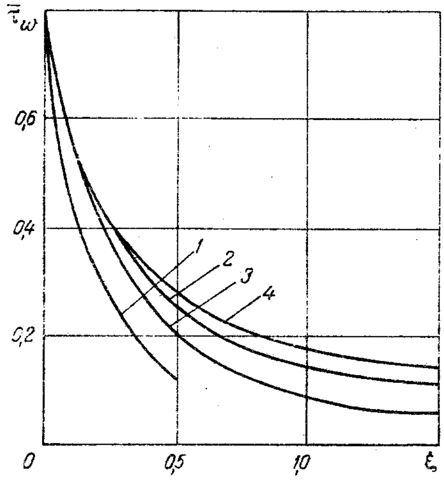

Fig. 2. Dimensionless shear stress on wall: 1) S = 1, B = 10; 2) S = 5, B = 1; 3) S = 10, B = 1; 4) S = 0, B = 1 (plate).

$$f = \frac{\partial f}{\partial \eta} = 0,\ \frac{\partial \theta}{\partial \eta} = -1 + \xi\theta(2 + B\xi\theta)(2 + 2B\xi\theta + B^2\xi^2\theta^2) \text{ for } \eta = 0,$$
$$\left(\frac{\partial f}{\partial \eta}\right) = \theta = 0 \text{ as } \eta \to \infty. \tag{8}$$

The parameter S defines the relative effect of radiation curvature on heat exchange. It may be regarded as the ratio of two characteristic lengths: the radiative length $2\lambda/(\sigma\varepsilon T_\infty^3)$ and the cylinder radius R. The ratio between the total thermal flux and the radiant component is determined by the complex B. The dimensionless longitudinal coordinate ξ establishes the relationship between radiant and convective components of the thermal flux.

We will note the characteristic peculiarities of Eqs. (6)-(8). At $\xi = 0$ they transform to the self-similar problem of free convection around a lamina with constant thermal flux [5]. At S = 0, Eqs. (6)-(8) lose their "cylindrical nature" and transform to the problem of interaction of radiation and free convection on a planar plate.

System (6)-(7) with boundary conditions (8) was calculated numerically by a finite-difference method. An implicit six-point finite-difference approach was used [6]. Derivatives with respect to ξ and η were approximated in the half-layer to second order accuracy. Nonlinear terms were linearized by the simple iteration method [7]. Calculations were performed for air, Pr = 0.7, and the following parameter values: S = 0, 1, 5, 10; B = 0, 1, 10.

Figure 1 presents profiles of dimensionless velocity $\bar{u} = \partial f/\partial\eta = u/[5(\nu/x)(Gr_x^*/5)^{2/5}]$ and temperature $\theta = (T - T_\infty)/[(q_w x/\lambda)(Gr_x^*/5)^{-1/5}]$ in the boundary layer for various values of S and ξ. With increase in ξ the maximum in dimensionless velocity shifts in the direction of larger η and decreases in value. The decrease in the velocity maximum occurs more rapidly than in the case of a planar plate (S = 0). The dimensionless temperature of the wall decreases insignificantly as compared to the planar plate case. However the reduction in temperature with transverse coordinate is more marked.

The dimensionless shear stress $\bar{\tau}_w = (\partial^2 f/\partial\eta^2)_0 = \tau_w/[(5\mu\nu/x^2)(Gr_x^*/5)^{2/5}]$ (Fig. 2) decreases with ξ more rapidly, the larger the radiation parameter B. Increase in the curvature parameter S leads to decrease in friction in comparison to the planar case.

Figure 3a, b presents curves of the change in local heat liberation from the cylinder surface in the forms

$$\frac{Nu_x}{Gr_x^{*1/5}} = -\frac{1}{5^{1/5}\theta_w}\left(\frac{\partial\theta}{\partial\eta}\right)_0, \tag{9}$$

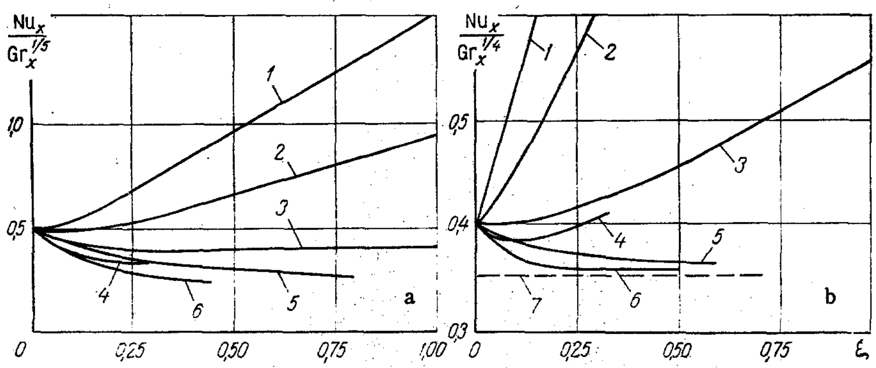

Fig. 3. Local heat liberation at wall versus longitudinal coordinates for: a) cylinder with q_w = const; b) cylinder with t_w = const: 1) S = 10, B = 1; 2) 5 and 1; 3) 1 and 1; 4) 1 and 10; 5) 0 and 1; 6) 0 and 10; 7) t_w = const (plate).

$$\frac{\mathrm{Nu}_x}{\mathrm{Gr}_x^{1/4}} = -\frac{1}{5^{1/4}\theta_w^{5/4}}\left(\frac{\partial\theta}{\partial\eta}\right)_0.$$

The opposing effects of radiation and transverse curvature on heat liberation can be seen. Radiation reduces, and curvature increases, heat liberation. At low ξ and B = 10 the effect of radiation appears more strongly due to the sharp drop in surface temperature. Such a decrease in heat liberation is noticeable only at $S \leq 1$. With further growth in ξ the influence of curvature becomes dominant, and heat liberation, having reached a minimum value, begins to increase. The position of the minimum shifts down the flow with increase in the radiation parameter B. At S = B = 1 interaction of curvature and radiation causes the heat liberation in the form $\mathrm{Nu}_x/\mathrm{Gr}_x^{*1/5}$ to maintain a constant value over a wide range of variation of ξ. For the case S = 0 (planar surface) heat liberation decreases with ξ.

In [1] it was shown that at large values of ξ the vertical planar radiating surface becomes isothermal. For a plate the local heat liberation in the form $\mathrm{Nu}_s/\mathrm{Gr}_x^{1/4}$ changes from 0.403 ($\xi = 0$, q_w = const) to 0.353 ($\xi \to \infty$, T = const). We will demonstrate that this is also valid for a vertical cylindrical surface. In Eqs. (1)-(4) we transform to variables used for an isothermal cylinder:

$$\eta_t = \left(\frac{\mathrm{Gr}_{x,\infty}}{4}\right)^{1/4}\frac{r^2-R^2}{2xR},\quad \xi_{t,\infty} = \frac{2x}{R}\left(\frac{\mathrm{Gr}_{x,\infty}}{4}\right)^{-1/4},$$
$$\psi = 4\nu\left(\frac{\mathrm{Gr}_{x,\infty}}{4}\right)^{1/4}\varphi(\xi_{t,\infty},\ \eta_t),\quad G(\xi_{t,\infty},\ \eta_t) = \frac{T-T_\infty}{\Delta T_\infty}, \tag{10}$$

where ΔT_∞ is a constant value subject to definition.

Boundary condition (4) for heat liberation from the surface is written in the form

$$S\frac{\Delta T_\infty}{T_\infty}\frac{1}{\xi_{t,\infty}}\left(\frac{\partial G}{\partial\eta_t}\right)_0 = \left(\frac{\Delta T_\infty}{T_\infty}G_w+1\right)^4 - 1 - B. \tag{11}$$

For $\xi_{t,\infty} \to \infty$ the left side of the equation tends to zero and $G_w \to 1$. From this condition we define ΔT_∞:

$$\left(\frac{\Delta T_\infty}{T_\infty}+1\right)^4 - 1 - B = 0 \text{ as } \xi_{t,\infty}\to\infty;\ \Delta T_\infty = T_\infty(\sqrt[4]{B+1}-1). \tag{12}$$

For small B, expanding the right side of Eq. (12) in a series, we obtain $\Delta T_\infty = q_w/4\sigma\varepsilon T_\infty^3$ [1]. The left side of Eq. (11) decreases with $\xi_{t,\infty}$ more rapidly, the smaller the curvature parameter S. Consequently, the function $T_w - T_\infty$ reaches its asymptotic value more rapidly at smaller S (Fig. 4). Curve 5 of Fig. 4 corresponds to heat liberation of an isothermal cylinder and is the asymptotic function of curves 1-4 for heat transfer with consideration of radiation. Approach to the asymptote occurs at smaller ξ_t with increase in B and decrease in S.

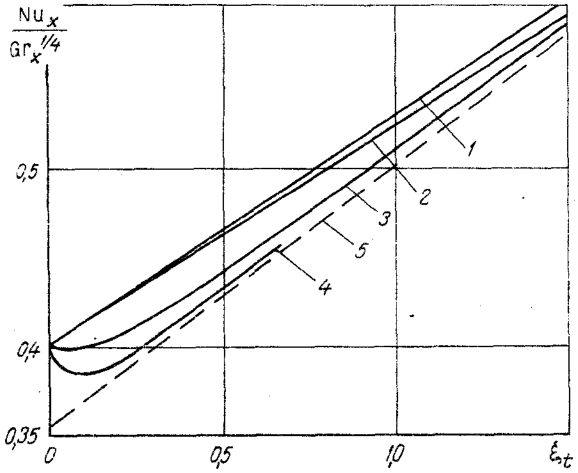

Fig. 4. Asymptotic values of local heat liberation at wall: 1) S = 10, B = 1; 2) 5 and 1; 3) 1 and 1; 4) 1 and 10; 5) $B \to \infty$ (T_w = const).

The variables ξ_t and ξ are related by the formula

$$\xi_t = \xi S \frac{1}{\theta_w^{1/4}} \left(\frac{4}{5}\right)^{1/4}, \tag{13}$$

while $\xi_t \to \xi_{t,\infty}$ as $\xi \to \infty$. The product $\xi S = 2x(Gr_x/5)^{-1/5}/R = \xi_q$ is the longitudinal variable problem of free convection on a cylindrical surface with constant thermal flux [5]. With use of the variables ξ_q, η in the boundary condition (8) for thermal flux on the wall there appears the new parameter $\Gamma = B/S = q_w R/(2\lambda T_\infty)$, not containing radiation characteristics:

$$\left(\frac{\partial\theta}{\partial\eta}\right)_0 = -1 + \frac{\Gamma}{B}\xi_q\theta_w(2 + \Gamma\theta_w\xi_q)(2 + 2\Gamma\theta_w\xi_q + \Gamma^2\theta_w^2\xi_q^2). \tag{14}$$

The limiting transition $B \to \infty$ ($\varepsilon = 0$, nonradiating surface) reduces boundary condition (14) to the form $-(\partial\theta/\partial\eta)_0 = 1$. Then in the variables η, ξ_q the problem describes free convective heat transfer of a vertical cylinder with q_w = const [5].

NOTATION

x and r, longitudinal and radial coordinates; u, v, velocity components along x and r axes; τ, shear stress; T, temperature; q, thermal flux; g, acceleration of gravity; β, μ, a, ν, λ, coefficients of volume expansion, dynamic viscosity, thermal diffusivity, kinematic viscosity, and thermal conductivity; σ, Stefan — Boltzmann constant; ε, surface emissivity; R, cylinder radius; ψ, flow function; $\Delta T_\infty = T_\infty(\sqrt[4]{B+1} - 1)$, temperature drop in boundary layer as $\xi \to \infty$; f, θ, φ, G, dimensionless flow function and temperature; $\eta = [(r^2 - R^2)/2Rx](Gr_x^*/5)^{1/5}$, dimensionless transverse coordinate; $\xi = \sigma\varepsilon T_\infty^3 x(Gr_x^*/5)^{1/5}/\lambda$, $\xi_q = 2x/R(Gr_x^*/5)^{-1/5}$, $\xi_t = 2x/R \cdot (Gr_x/4)^{-1/4}$, $\xi_{t,\infty} = 2x/R(Gr_{x,\infty}/4)^{-1/4}$, dimensionless longitudinal variables; $Gr_x = g\beta(T_w - T_\infty)x^3/\nu^2$, $Gr_{x,\infty} = g\beta\Delta T_\infty x^3/\nu^2$, Grashof criteria; $Gr_x^* = g\beta q_w x^4/(\lambda\nu^2)$, modified Grashof criterion; $Nu_x = \alpha x/\lambda$, Nusselt number; $Pr = \nu/a$, Prandtl number; $B = (q_w/\sigma\varepsilon T_\infty^4)$, $\Gamma = q_w R/(2\lambda T_\infty)$, $S = 2\lambda/(\sigma\varepsilon R T_\infty^3)$, dimensionless parameters. Indices: x, local value; w, wall; 0, body surface; ∞, asymptotic value; t, specified temperature; q, specified thermal flux.

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